

WHEN DOES A CRACK GROW UNDER MODE II CONDITIONS?

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Abstract

Even cracks that seem to be favourably oriented for mode II growth might extend in mode I by kinking. The conditions that favour mode I or mode II growth are examined. It is shown that a high confining pressure promotes mode II growth and so does a low ratio between the critical stress intensity factors of modes II and I. Quantitative results are given for different coefficients of friction and confining pressures.

1. Introduction

Attempts to realize mode II crack propagation on a laboratory scale usually fail because a mode I growth takes over. On the other hand, traces of mode II growth are often detected at or after earth-quake slipping. It is generally anticipated that a high confining pressure promotes mode II growth. It has also been suggested that some materials (such as rocks) are more susceptible to fracture under mode II conditions than other materials (such as metals), see e.g. [1]. In the present paper the conditions for mode II instead of mode I growth are investigated with respect both to confining pressure and to pertinent material characteristics.

It is well known that mode II growth - if it appears - proceeds in a direction that forms an angle to the direction of maximum shear stress if frictional sliding takes place. This angle is found as the one that implies maximum mode

II stress intensity factor K_{II} , which also implies vanishing mode I stress intensity factor, K_I . Mode I growth - if it appears - will proceed in another direction. This direction is assumed to be the one for which K_I is maximum.

Plane cases (for instance plane strain) are studied. The preferred angle for mode II crack growth can then be found analytically. It is assumed that a straight crack (or a fault segment) in this direction exists. If mode II growth appears the crack will be extended without directional change, but if mode I takes over, the crack will be extended via a kink. The mode I stress intensity factor at the tip of the kink can be determined by means of numerical methods. A certain kink angle gives a maximum $K_I = K_{I\max}$. As mentioned previously this is assumed to be the direction for mode I growth - if such growth appears.

2. Preferred direction for mode II growth

Consider a crack of length $2a$, subjected to a pressure p and a shear stress $\tau > 0$. The crack is assumed to form an angle θ to the direction of the shear stress according to Fig. 1. By pressure is meant the mean in-plane stress with reversed sign. Plane stress or plane strain is assumed. The mode II stress intensity factor is given by the expression

$$K_{II}/\sqrt{\pi a} = \tau \cos 2\theta - \tau_n \quad (1)$$

where τ_n is the shear stress on the crack surfaces.

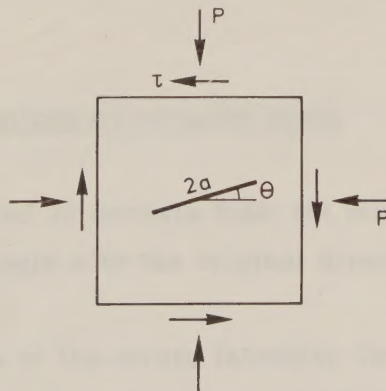


Fig. 1. Crack configuration.

Assuming Coulomb friction with friction coefficient μ , slip takes place when

$$\tau_n = -\mu\sigma_n \quad (2)$$

where σ_n is the normal stress on the crack surfaces, i.e.

$$\sigma_n = \begin{cases} 0 & \text{if } p/\tau \leq \sin 2\theta \\ -p + \tau \sin 2\theta & \text{if } p/\tau > \sin 2\theta \end{cases} \quad (9)$$

Then the expression for K_{II} reads

$$K_{II}/\sqrt{\pi a} = \begin{cases} \tau \cos 2\theta & \text{if } p/\tau \leq \sin 2\theta \\ -\mu p + \tau (\cos 2\theta + \mu \sin 2\theta) & \text{if } p/\tau > \sin 2\theta \end{cases} \quad (4)$$

The maximum value of K_{II} appears for

$$\theta = \theta_d = \begin{cases} 0 & \text{if } p/\tau < 0 \\ \frac{1}{2} \sin^{-1}(p/\tau) & \text{if } 0 < p/\tau \leq \mu/\sqrt{1+\mu^2} \\ \frac{1}{2} \tan^{-1} \mu & \text{if } \mu/\sqrt{1+\mu^2} < p/\tau < \sqrt{1+\mu^2}/\mu \end{cases} \quad (5)$$

A crack originally situated in this direction will grow in mode II without change of direction with the stress intensity factor [1]:

$$K_{II\max}/\sqrt{\pi a} = \begin{cases} \tau & \text{if } p/\tau < 0 \\ \sqrt{\tau^2 - p^2} & \text{if } 0 < p/\tau \leq \mu/\sqrt{1+\mu^2} \\ -\mu p + \tau \sqrt{1+\mu^2} & \text{if } \mu/\sqrt{1+\mu^2} < p/\tau < \sqrt{1+\mu^2}/\mu \end{cases} \quad (6)$$

provided that $K_{II\max}$ reaches the critical mode II stress intensity factor K_{IIc} and that mode I growth for some reason is prevented.

3. Finite element calculations for straight kinks

The crack is now assumed to deviate from its straight path and be extended via a kink, forming an angle α to the original direction of growth, see Fig. 2.

Numerical determination of the stress intensity factors from singular integral equations was first considered. However, although this method is superior to finite element methods for smoothly curved cracks, it appears to be inferior for kinked cracks [2]. Therefore finite element methods were chosen.

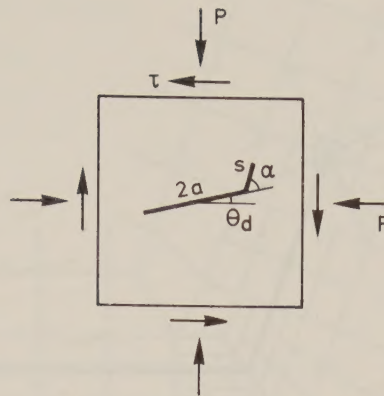


Fig. 2. Kinked crack configuration.

The stress intensity factors at the kink tip were calculated from the displacement field, using the finite element program ABAQUS [3]. Eight node isoparametric plane strain elements were chosen with the mid side nodes in the elements surrounding the crack tips moved to a quarter point of each element side, whereby a square root singular deformation field at the crack tip is simulated [4]. Between the crack surfaces interface elements were included in order to simulate Coulomb type friction whenever the crack surfaces are in contact. The total number of elements was 462, the number of nodes was 1578 and the number of degrees of freedom was 3153. Reduced integration was used. The mesh in the vicinity of the kink is shown in Fig. 3. The whole mesh occupies a square with side length $30a$. Thus a 'large' region is studied, in fact large enough that the stress intensity factors for the straight crack differ less than one per cent from those obtained for an infinite region.

A check on the accuracy is obtained by calculating K_{II} for a straight crack and $\mu = 0$. The result indicates that the accuracy is well within the one per cent level.

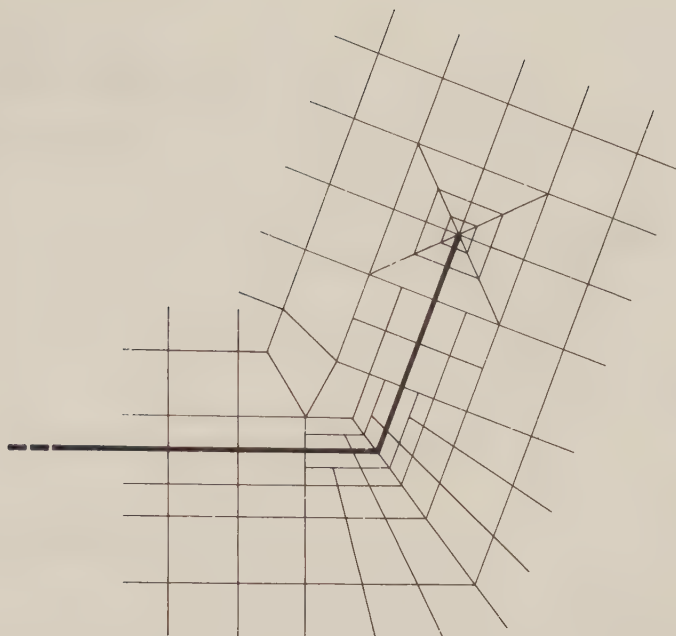


Fig. 3. Finite element mesh in the vicinity of the kink.

The stress intensity factors are calculated from the displacement field at the quarter point nodes by assuming three different contributions:

- 1) The square root mode I and II displacement fields.
- 2) A superimposed constant strain field ϵ_{x0} parallel to the direction of the kink.
- 3) A rigid body rotation ω of the region surrounding the kink tip.

Then the displacements relative to the kink tip can be written

$$u_x - u_{x0} = \frac{K_I}{8\pi G} \sqrt{2\pi r} \left[(2\kappa - 1) \cos \frac{\phi}{2} - \cos \frac{3\phi}{2} \right] + \epsilon_{x0} r \cos \theta \\ + \frac{K_{II}}{8\pi G} \sqrt{2\pi r} \left[(2\kappa + 3) \sin \frac{\phi}{2} + \sin \frac{3\phi}{2} \right] - r \omega \sin \phi \quad (7)$$

$$u_y - u_{y0} = \frac{K_I}{8\pi G} \sqrt{2\pi r} \left[(2\kappa + 1) \sin \frac{\phi}{2} - \sin \frac{3\phi}{2} \right] - \frac{3 - \kappa}{1 + \kappa} \epsilon_{x0} r \sin \phi \\ - \frac{K_{II}}{8\pi G} \sqrt{2\pi r} \left[(2\kappa - 3) \cos \frac{\phi}{2} + \cos \frac{3\phi}{2} \right] + r \omega \cos \phi \quad (8)$$

where (x, y) and (r, ϕ) refer to the local coordinate systems at the kink tip, see Fig. 4. u_{x0} and u_{y0} are the displacements of the kink tip and

$$\kappa = \begin{cases} 3-4\nu & \text{in plane strain} \\ (3-\nu)/(1+\nu) & \text{in plane stress} \end{cases}$$

where ν is Poisson's ratio.

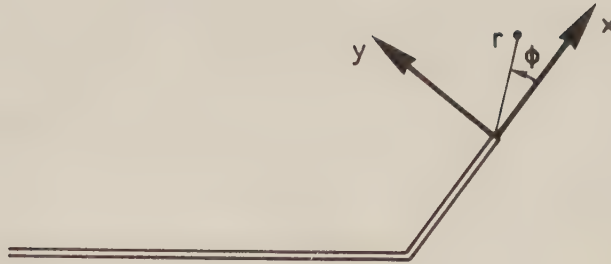


Fig. 4. Local coordinate systems at the kink tip.

Both equations (7) and (8) consist of a symmetric and an antisymmetric part with respect to the coordinate ϕ . Consider two nodes i and j , symmetrically situated with respect to the x -axes, i.e. $r_j = r_i$, $\phi_j = -\phi_i$. By forming the sum of and difference between their displacements $(u_{xi}-u_{xo}, u_{yi}-u_{yo})$ and $(u_{xj}-u_{xo}, u_{yj}-u_{yo})$ according to equations (7) and (8) one obtains

$$\begin{aligned} \frac{1}{2}(u_{xi}+u_{xj})-u_{xo} &= \frac{K_I}{8\pi G}\sqrt{2\pi r_i} \left[(2\kappa-1)\cos\frac{\phi_i}{2} - \cos\frac{3\phi_i}{2} \right] \\ &\quad + \epsilon_{xo} r_i \cos\phi_i \\ \frac{1}{2}(u_{xi}-u_{xj}) &= \frac{K_{II}}{8\pi G}\sqrt{2\pi r_i} \left[(2\kappa+3)\sin\frac{\phi_i}{2} + \sin\frac{3\phi_i}{2} \right] \\ &\quad - r_i \omega \sin\phi_i \\ \frac{1}{2}(u_{yi}+u_{yj})-u_{yo} &= -\frac{K_{II}}{8\pi G}\sqrt{2\pi r_i} \left[(2\kappa-3)\cos\frac{\phi_i}{2} + \cos\frac{3\phi_i}{2} \right] + \\ &\quad + r_i \omega \cos\phi_i \\ \frac{1}{2}(u_{yi}-u_{yj}) &= \frac{K_I}{8\pi G}\sqrt{2\pi r_i} \left[(2\kappa+1)\sin\frac{\phi_i}{2} - \sin\frac{3\phi_i}{2} \right] - \\ &\quad - \frac{3-\kappa}{1-\kappa} \epsilon_{xo} r_i \sin\phi_i \end{aligned} \quad (9)$$

From the displacements obtained by the finite element calculation in the quarter point nodes and at the crack tip, the four unknown quantities K_I , K_{II} ,

ϵ_{x0} and ω are calculated.

The variation of the stress intensity factors at the kink tip with the ratio s/a between kink length s and original half-length a of the crack is shown in Fig. 5 for $\mu = 0.7$ and $p/\tau = 1$ and for a few kink angles near those for which K_I is maximum and $K_{II} = 0$.

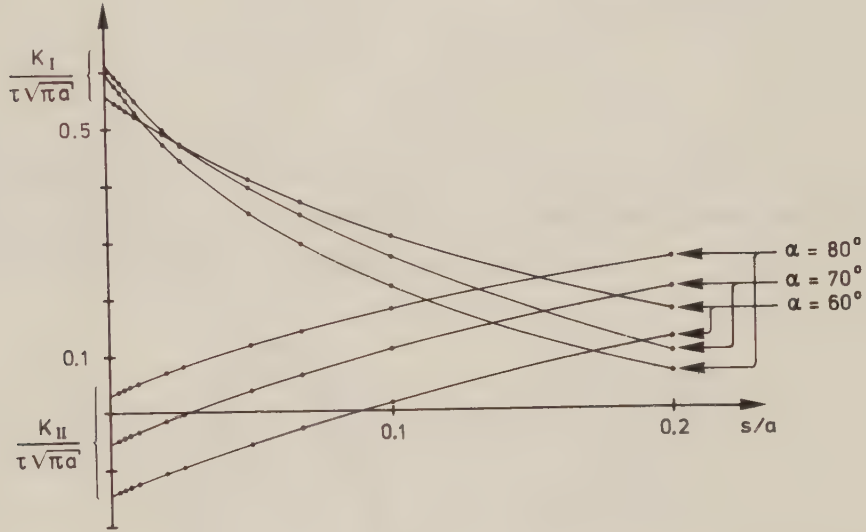


Fig. 5. Variation of the stress intensity factors at the kink tip as a function of the ratio s/a between kink length s and original half length a of the crack for different values of the kink angle α . $\mu = 0.7$ and $p/\tau = 1$.

4. Incipient kinking

As can be seen from Fig. 5, K_I and K_{II} appear to approach limiting values asymptotically for each kink angle as $s/a \rightarrow 0$. This observation is strongly supported by a study of Figs. 6 and 7. In Fig. 6 a typical contour of the separated surfaces is shown. It is noticed that separation occurs along a portion of the original crack near the kink as well as at the kink itself. In Fig. 7 the separation of the kink surfaces for kink angle $\alpha = 70$ degrees, $\mu = 0.7$ and $p/\tau = 1$ is shown for different s/a . With the scales used the kink contours nearly coincide and apparently tend towards a limiting contour as $s/a \rightarrow 0$. It is possible to show that this implies nearly constant mode I stress intensity factor, tending towards a limiting value as $s/a \rightarrow 0$.



Fig. 6. Separation of crack and kink surfaces for $s/a = 0.004$, $\alpha = 70$ degrees, $\mu = 0.7$ and $p/\tau = 1$. (Exaggeration of the scale gives the illusion that shear strains are very large at the corners.)

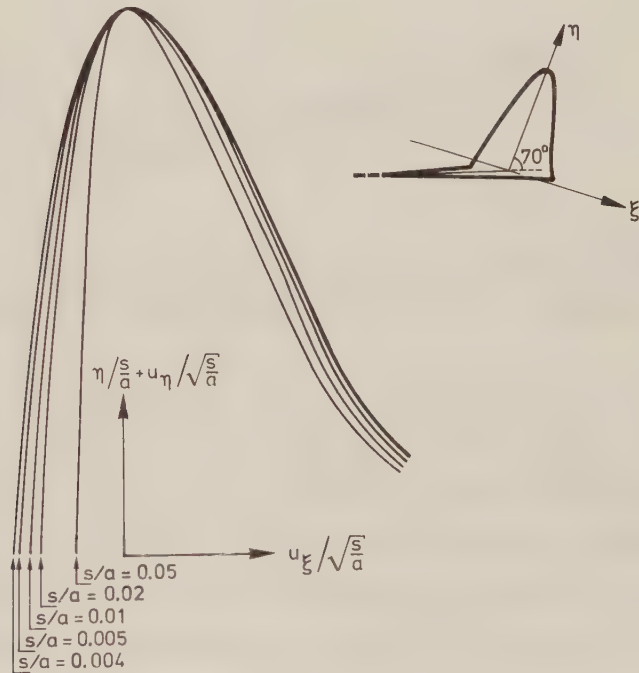


Fig. 7. Separation of the kink surfaces for $\alpha = 70$ degrees, $\mu = 0.7$ and $p/\tau = 1$.

The direction of incipient mode I growth is assumed to be the one that maximizes the mode I stress intensity factor for given stress state and coefficient of friction. Ideally, of course, this direction should be found for infinitesimally small s/a . However, the asymptotic behaviour anticipated from a study of Figs. 5 and 7 suggests that a very small error results if s/a is

chosen to be smaller than about 0.01. In the present investigation the value $s/a = 0.004$ is chosen to represent an infinitesimally small kink.

The variation of K_I and K_{II} with the kink angle α at the kink tip for $s/a = 0.004$ in the neighbourhood of the maximum of K_I is shown in Fig. 8 for $\mu = 0.4$ and $\mu = 0.7$, both curves with $p/\tau = 1$.

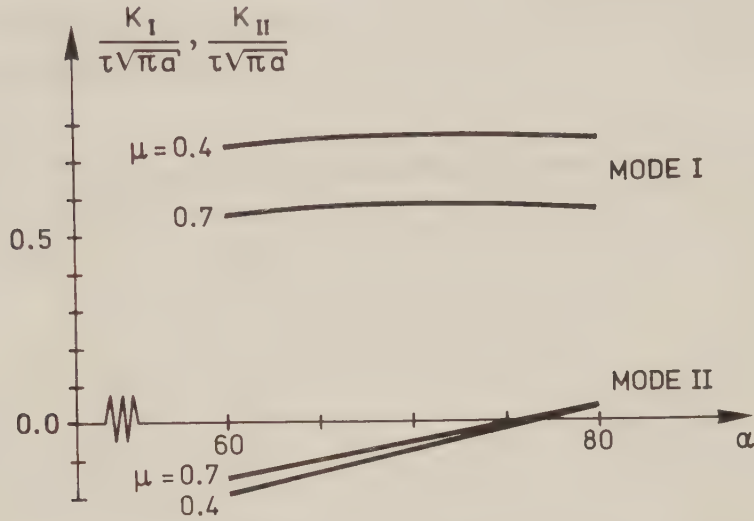


Fig. 8. Variation of K_I and K_{II} at the kink tip with kink angle α in the neighbourhood of the maximum of K_I for $p/\tau = 1$ and $s/a = 0.004$.

It is seen that K_{II} is close to zero as the maximum of K_I is reached. However, it has not been possible to show that K_{II} should vanish when K_I reaches maximum for incipient kinking. The results obtained rather indicate that this is not the case in general. During continued kink growth, however, the crack can be regarded as straight in a sufficiently close vicinity of the crack tip and it is known that the relations $K_{II} = 0$ and $K_I = K_{I\max}$ occur simultaneously for straight cracks.

From results such as those shown in Fig. 8 it is estimated that incipient kinking occurs at an angle of about 70 degrees to the main crack. This holds for all investigated combinations of p/τ and μ . Naturally, a necessary condition for incipient kinking is that K_I reaches the critical mode I stress intensity factor K_{IC} . In addition, of course, the situation must be such that mode I is the preferred mode, a question that will be discussed later.

It is worth noticing that the maximum of σ_ϕ before kinking occurs at the angle 70.5 degrees (approximately).

As the ratio p/τ is increased the value of $K_{I\max}$ decreases whereas K_{II} apparently increases, see Fig. 9, which also shows that the values of $K_{I\max}$ become independent of $\mu > \mu_0$ for $p/\tau < \mu_0/\sqrt{1+\mu_0^2}$. This is a consequence of the fact that the angle θ_d (determining the choice of original crack direction) is independent of μ in this region, cf. (5), and that the crack is open so that no frictional sliding is involved.

The variation of $K_{I\max}$ and K_{II} with the coefficient of friction for $p/\tau = 1$ and $p/\tau = 1.5$ is shown in Fig. 10.

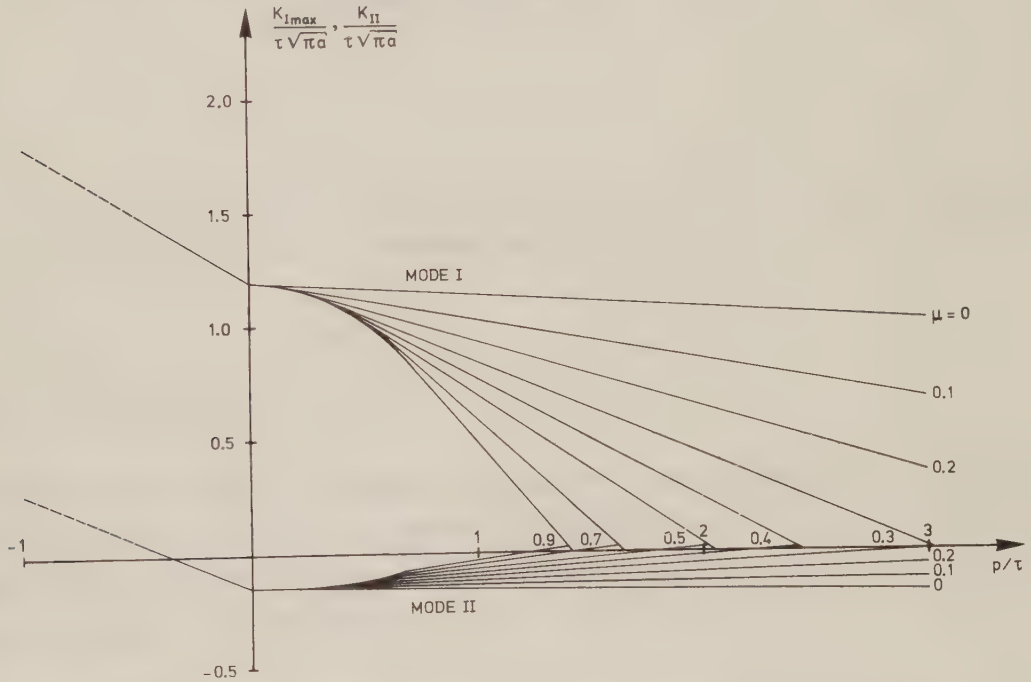


Fig. 9. Stress intensity factors K_I and K_{II} at the kink tip versus p/τ for varying coefficient of friction μ and for kink angle $\alpha = 70$ degrees, i.e. about the angle for which K_I is maximum. Thus $K_I \approx K_{I\max}$. The diagram is based on 42 numerical calculations of K_I and K_{II} . The resulting diagram points lie very close to the straight lines drawn - the deviation is less than 0.005 in ordinate measure. For p/τ less than about -0.5 the lines are dashed to indicate that the largest K_I is not obtained for a kinked but for a straight crack, cf. Fig. 12.

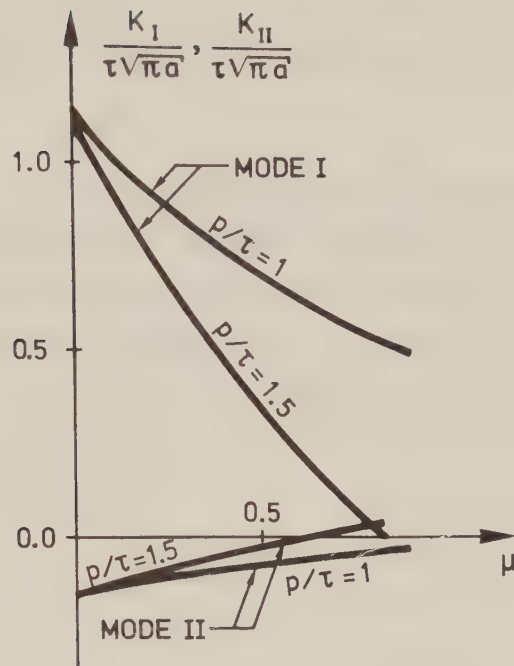


Fig. 10. Variation of K_I and K_{II} at the kink tip with μ for $p/\tau = 1$ and $p/\tau = 1.5$. Kink angle $\alpha = 70$ degrees and $s/a = 0.004$.

5. Kink growth and arrest

If the crack after incipient kinking continues to grow in mode I it will follow some smooth path and, if the growth continues far enough, it will tend towards the direction perpendicular to the largest principal stress, i.e. towards kink angle $\alpha = \pi/4 - \theta_d$.

The extension of the kink along a curved path is modelled by piece-wise straight segments in four steps, according to Fig. 11. Initial kink angle α_1 is 70 degrees in agreement with the result for an 'infinitesimal' kink. In the following three steps α_2 , α_3 and α_4 are determined from the condition $K_{II} = 0$. This gives a more precise determination of the kink angle for larger values of s/a than the condition $K_I = \text{maximum}$. The value of K_I during the growth is also shown in Fig. 11, including the value of K_I for $s/a = 0.004$ (cf. Fig. 5). Furthermore a smooth curve is drawn through the points obtained and 'manual' extrapolation to $s/a = 0$ and to $K_I = 0$ is made. One observes that a kink is opened if $K_{Ic}/\tau\sqrt{s/a}$ is less than about 0.3, provided that mode II

growth does not take place instead. Even with a very small K_{IC} the kink cannot reach a length of about $0.05a$ (except possibly under very special dynamic conditions).

The determination of K_I was made under the assumption that p/τ remained constant during kink growth. Another situation, for instance kink growth under increasing τ while p is kept constant, would not change the crack path, but it would influence the magnitude of K_I . Thus it would be possible, for instance, to adjust τ at constant p so that $K_I = K_{IC}$ during kink growth until global instability occurs. For $p/\tau < 0$ global instability seems to occur immediately after kink formation.

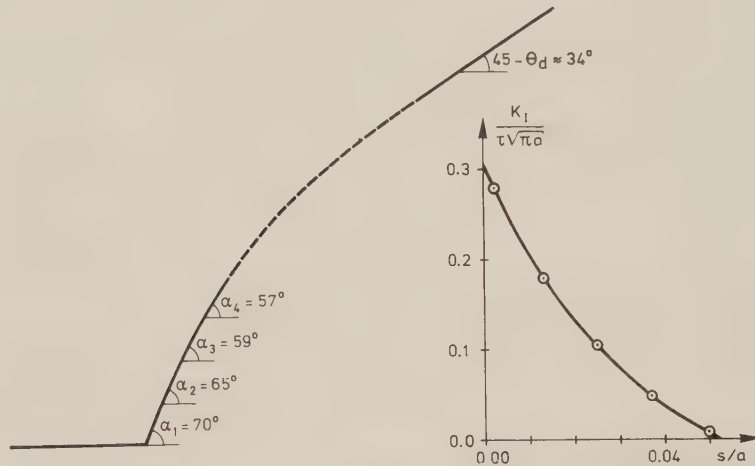


Fig. 11. Extension of the kink along a curved path. $\mu = 0.4$ and $p/\tau = 1.9$. s is the arc-length along the kink. The K_I -curve depends critically on p/τ : for $p/\tau < 0$ K_I appears to increase with s/a .

6. Which mode will be preferred?

A criterion for deciding whether the crack will grow in mode I by formation and extension of a kink or grow in mode II in the original crack direction might be formulated by comparison of the stress intensity factors $K_{I\max}$ for a small kink and $K_{II\max}$ for the main crack, according to (6). If K_{IC} and K_{IIC} denote the critical stress intensity factors for mode I and II, respectively, mode II growth will be preferred to mode I, if

$$\kappa = \frac{K_{II\max}}{K_{I\max}} > \frac{K_{IIc}}{K_{Ic}} = \kappa_c \quad (11)$$

In Fig. 12 the ratio κ is shown as a function of p/τ for $\mu = 0.4$. The limits for mode II growth, $K_{II\max} = 0$, cf. eq. (6), and mode I growth, $K_{I\max} = 0$, cf. Fig. 9, are also shown.

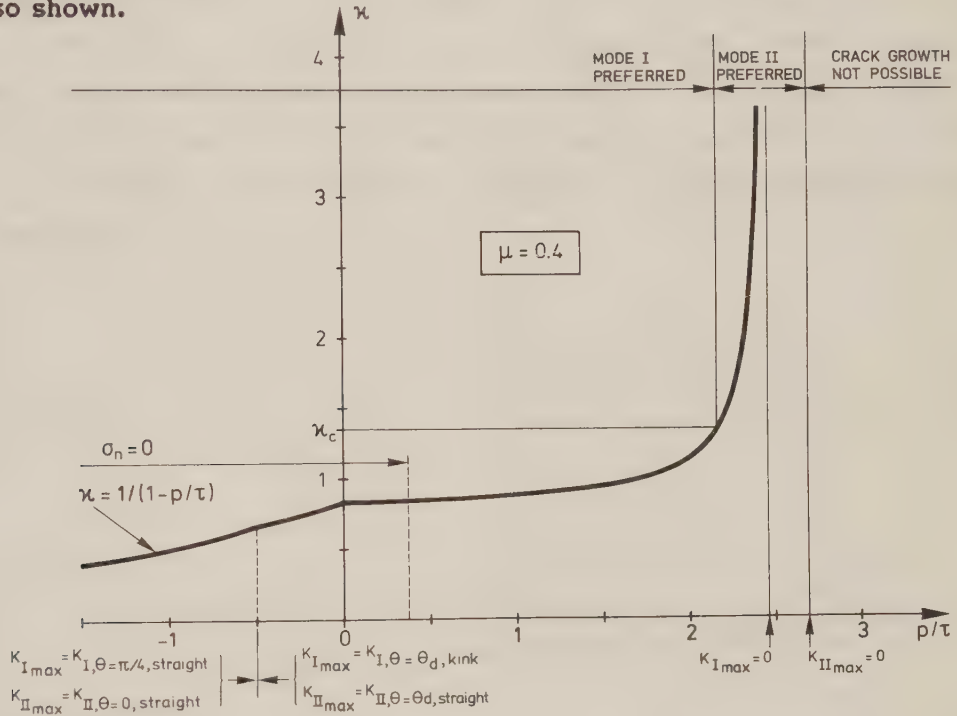


Fig. 12. κ versus p/τ for $\mu = 0.4$. Kink angle $\alpha = 70$ degrees. The remarks 'Mode I preferred' or 'Mode II preferred' do not imply that mode I or mode II growth actually takes place: the necessary condition $K_{I\max} \geq K_{Ic}$ or $K_{II\max} \geq K_{IIc}$ must be added.

For $p/\tau < 1$ the normal stress σ_n at the crack surfaces vanishes for a crack perpendicular to the direction of the largest principal stress. The mode I stress intensity factor $K_I = (1-p/\tau)\tau\sqrt{\pi a}$. As can be seen from Fig. 9 this stress intensity factor is lower than the maximum mode I stress intensity factor at the tip of the kink for p/τ larger than about -0.5 . For lower p/τ , however, the largest possible mode I stress intensity factor at incipient growth of a crack of length $2a$ is found for a crack direction perpendicular to the direction of the largest principal stress. Thus $K_{I\max} = (1-p/\tau)\tau\sqrt{\pi a}$ for p/τ less than about -0.5 . Still $K_{II\max}$ is given by (6), i.e. $K_{II\max} = \tau\sqrt{\pi a}$, albeit this maximum occurs for a crack with another orientation, viz. parallel to the direction of the largest shear stress. Hence one can put

$\kappa = K_{II\max}/K_{I\max} = 1/(1-p/\tau)$ for p/τ less than about -0.5 . These values of κ are regarded to be the most pertinent ones for a criterion about which mode will be preferred. Therefore they are used in Fig. 12.

One observes that κ is rather close to unity for p/τ around zero but increases towards infinity with increasing p/τ . Thus, in the region where $K_{I\max} > 0$, the ratio κ will reach any predetermined value if p/τ is sufficiently high. Mode II crack growth from the tip of the original crack is preferred to kinking and mode I growth whenever the value of p/τ is such that $\kappa > \kappa_c$, whereas mode I growth by extension of a kink is preferred for lower values of p/τ .

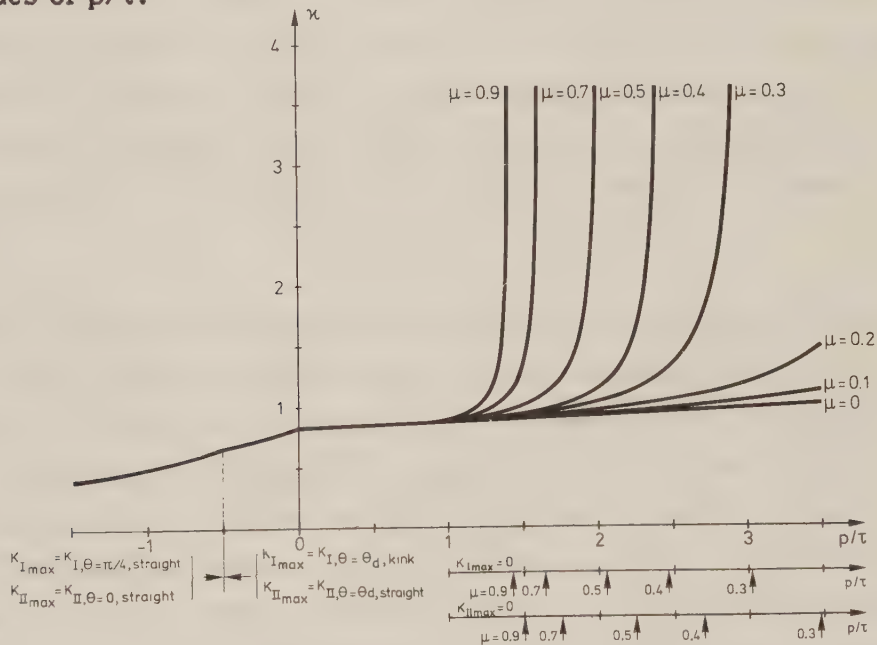


Fig. 13. κ versus p/τ for different values of μ . Kink angle $\alpha = 70$ degrees.

For $\mu = 0.1$ $K_{I\max} = 0$ for $p/\tau \approx 6.8$ and $K_{II\max} = 0$ for $p/\tau \approx 10.1$ For $\mu = 0$ $K_{I\max} = 0$ for $p/\tau \approx 19.7$ and $K_{II\max} \rightarrow 0$ as $p/\tau \rightarrow \infty$

Fig. 13 shows κ versus p/τ for different values of the friction coefficient. The limits for mode I and mode II growth are also included. For each μ the κ -values coincide with those for all higher values of $\mu > \mu_0$ when $p/\tau < \mu_0/\sqrt{1+\mu_0^2}$. However, this can not be seen in Fig. 13, because the largest possible meaningful resolution (which depends on the accuracy) is not high enough.

7. Discussion

Whereas it was possible to determine the value of $K_{I\max}$ quite accurately, the kink angle at which this maximum occurs could not be determined so well. However, this does not influence the results presented here, since this angle is of secondary significance.

The investigation confirms the anticipation referred to in the introduction that a high confining pressure promotes mode II growth. Furthermore it seems to be possible to describe the role of the material by the ratio $\kappa_c = K_{IIc}/K_{Ic}$. If κ_c is around 0.82 or smaller, mode II seems to be preferred whenever there is a confining pressure. If κ is around unity or larger, mode I is preferred if the pressure is smaller than the shear stress (for the range of friction coefficients investigated, $\mu = 0 - 0.9$). In such cases the range of p/τ for which mode II is preferred is rather small. This can, for instance, be seen from Fig. 12.

In a typical development towards earth-quake slip the confining pressure p is fairly constant, whereas the shear stress τ increases with time. Thus p/τ will decrease and therefore the region in which mode II is preferred will be reached before the corresponding region for mode I, see Fig. 12. If the confining pressure p is high enough $K_{II\max}$ will reach K_{IIc} before τ has reached the level at which mode I is preferred, even if the ratio κ_c is comparatively large. Then a mode II slip event will occur, assuming, of course, the pre-existence of a fault segment in a favourable direction for mode II growth.

In situations more typical for engineering, mode II failures seem to be rare. Together with the fact that high confining pressures also are rare, this indicates that the ratio κ_c is around unity or larger for most engineering materials. A consequence is that an experimental determination of K_{IIc} needs a high confining pressure, something that appears to imply severe experimental difficulties.

It should be observed that the nature of the conditions for mode II directional stability is different from the one for mode I directional stability [5]. In the latter case material characteristics are insignificant.

Non-linear effects have been disregarded. Therefore the results obtained are not necessarily applicable to cases of small cracks, for instance.

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